SELECTION OF MIXED SAMPLING PLAN WITH CSP-3(K=I) PLAN AS ATTRIBUTE PLAN INDEXED THROUGH MAPD AND MAAOQ

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ABSTRACT:

In this paper a procedure for the construction and selection of the independent mixed sampling plan using MAPD and MAAOQ as quality standards with Continuous Sampling plan of the type CSP-3(k=i) as attribute plan is presented. Tables are constructed for the selection of parameters of the plan when MAPD and MAAOQ are given. Practical applications of the sampling plan are also discussed with suitable example.

Key words and Phrases: *Maximum allowable percent defective, Maximum allowable average outgoing quality, Operating characteristic.*

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1. Introduction:

A variety of plans and procedures have been developed for special sampling situation involving both measurements and attributes. Each is tailored to do a specific job under prescribed circumstances. They range from a simplified variables approach to a more technically complicated combination of variables and attribute sampling called Mixed sampling plans.

Mixed sampling plans are of two types, namely independent and dependent plans. Independent mixed plans do not incorporate first sample results in the assessment of the second sample. Dependent mixed plans combine the results of the first and second samples in making a decision if a second sample is necessary.

Mixed sampling plans consist of two stages of rather different nature. During the first stage the given lot is considered as a sample from the respective production process a criterion by variables is used to check process quality. If process quality is judged to be sufficiently good, the lot is accepted, otherwise the second stage of the sampling plan is entered and lot quality is checked directly by means of an attribute sampling plan.

Mixed Sampling Plan (MSP) was first developed by Schilling (1967) for the case of single sided specifications, standard deviation known by assuming an underlying normal distribution for measurements. Dodge (1943) provided the concept of continuous sampling inspection and introduced the first continuous sampling plan, originally referred to as the random – order and later designated as CSP-1 plan by Dodge and Torrey (1951). Dodge (1947) outlined several sampling plans for continuous production. Kandasamy (1993) studied in designing of various types of continuous sampling plans. Suresh and Ramkumar (1996) discussed about the use of MAAOQ for the selection of sampling plans. Radhakrishnan (2002) constructed continuous sampling plans indexed through MAAOQ and mentioned its advantage over AOQL. Devaarul (2003), Sampath Kumar (2007), Radhakrishnan and Sampath Kumar (2006a, 2006b, 2007a, 2007b, 2007c, 2009), Radhakrishnan et.al (2010) have made contributions to mixed sampling plans for independent case. Radhakrishnan et.al and (2009) studied mixed sampling plan for dependent case.

In this paper, using the operating procedure of mixed sampling plan (independent case) with CSP-3(k=i) as attribute plan, tables are constructed for the mixed sampling plan indexed through MAPD and MAAOQ. Suitable suggestions are also provided for the future.

2. Glossary of Symbols:

The symbols used in this paper are as follows

p : submitted quality of lot or process

p_* : maximum allowable percent defective (MAPD)

β_j : probability of acceptance for lot quality 'p_j'

β'_j : probability of acceptance assigned to first stage for percent defective 'p_j'

β''_j : probability of acceptance assigned to second stage for percent defective 'p_j'

k : variable factor such that a lot is accepted if $\bar{X} \geq A = L + k\sigma$

f : the rate of inspection(=1/n)

i : number of consecutive units are found conforming

n_1 : sample size for the variable sampling plan

n_2 : sample size for the attribute sampling plan = (1/f) units

3. Formulation of Mixed Sampling Plan with CSP-3(k=i) as Attribute Plan:

The development of mixed sampling plans and the subsequent discussions are limited only to the Lower specification limit 'L'. By symmetry a parallel discussion can be used for upper specification limits also. It is suggested that the mixed sampling plan with CSP–3(k=i) in the case of single sided specification (L) , standard deviation (σ) known can be formulated by the parameters i, n_1, n_2 and k.

Mixed sampling procedure suggested by Schilling (1967) is slightly modified and presented in this paper and this procedure is to be adopted separately for each time period fixed by the manufacturer. By giving the values for the parameters an independent plan for single sided specification, σ known would be carried out as follows:

- Determine the parameters with reference to ASN and OC curves
- Take a random sample of size n_1 from the lot assumed to be large during the time period 't' (may be an hour / a shift / a day / a week …). [This is the modification suggested in this paper over Schilling (1967)]
- If a sample average $\bar{X} \geq A = L + k\sigma$, accept the lot

❖ If the sample average $\bar{X} < A = L + k\sigma$, apply the operating procedure of CSP-3(k=i)

Operating Procedure Of CSP-3(k=i) Plan

Step l: Specify f and i.

Step 2: Begin 100% inspection.

Step 3: After i units in succession have been found without a defective, start sampling inspection.

Step 4: Randomly inspect a fraction f of the units.

Step 5: When a defective is found, inspect the next 4 units, if an additional defective is found revert to 100% inspection otherwise, continue sampling for k successive sample units. If no defectives are found in k, continue sampling by selecting a fraction f of the units. If a defective is found in the k samples revert to 100% inspection immediately.

To reduce the number of parameters from three to two, it is assumed k=i. So CSP-3(k=i) is a particular type of sampling plan of CSP-3 with k=i.

4. Construction and Selection of Mixed Sampling Plan having CSP-3(k=i) as attribute plan indexed through MAPD and MAAOQ:

Maximum allowable percent defective (MAPD) is the quality level that corresponds to the point of inflection of the OC curve. It is the quality level at which the second order derivative of the OC function $P_a(p)$ with respect to p is zero. MAPD is used as an index for acceptance sampling plans. When some specific value for a characteristic or group of characteristics is designated, the continuous sampling plan will have a tendency to accept product during periods of sampling if the submitted quality is upto MAPD and if the submitted quality is beyond MAPD, the sampling plan will have a tendency to submit the product for screening. The inflection point (p_*) is obtained by using $d^2P_a(p)/dp^2 = 0$ and $d^3P_a(p)/dp^3 \neq 0$.

Maximum allowable average outgoing quality (MAAOQ) of a sampling plan is designated as the Average Outgoing Quality(AOQ) at the MAPD.

AOQ = p. $P_a(p)$. MAAOQ = AOQ at p = p_*.

Schilling (1967) has given a procedure for constructing the mixed sampling plan when a point on OC curve and n_1 are known. With CSP-3(k=i) as attribute plan to satisfy (p_*, β_*), n_1, n_2 and i on the OC curve, the procedure is given as follows:

- Assume that the mixed sampling plan is independent.
- Split the probability of acceptance (β_j) determining the probability of acceptance that will be assigned to the first stage. Let it be β_j'.
- Decide the sample size n_1 (for variable sampling plan) to be used.
- Calculate the acceptance limit for the variable sampling plan as

  $A = L + k\sigma = L + [z(p_j) + \{z(\beta_j')/\sqrt{n_1}\}]\sigma$, Where z(t) is the standard normal variate corresponding to 't' such that $t = \int_{z(t)}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-u^2/2} du$.
- Determine the sample average $\overline{X}$. If a sample average $\overline{X} < A = L + K\sigma$, take a second stage sample of size 'n_2' using attribute sampling plan.
- Now determine β_j'' the probability of acceptance assigned to the attributes plan associated with the second stage as $\beta_j'' = \frac{\beta_j - \beta_j'}{1 - \beta_j'}$.
- Determine the values of n_2 and i from Pa(p)= β_j'' for p = p_j.

Using the above procedure tables can be constructed to facilitate easy selection of mixed sampling plan with CSP-3(k=i) as attribute plan indexed through MAPD and MAAOQ.

Construction of Tables

According to Stephens (1979), the OC function of the CSP-3(k=i) plan is given by

$$P_a(p) = \frac{q^i[1 + q^4(1 - q^i)]}{f[1 - q^i - q^{i+4}(1 - q^i)] + q^i[1 + q^4(1 - q^i)] + 4pfq^i}$$

Where p is the fraction of incoming lots that are not acceptable and q=1-p and f=1/n. The values of i and n are calculated for different possible combinations of MAPD and MAAOQ for β_*' = 0.20 using Visual Basic Program and presented in Table 1.

Selection of the Plan

For the specified values of MAPD and MAAOQ one can find the ratio MAPD / MAAOQ and select the nearest value of the ratio MAPD/MAAOQ in Table 1 and corresponding to the given value of MAPD, the values of i and n_2 are obtained from Table 1.

Example : 1

Given MAPD = 0.0015 and MAAOQ = 0.000599. Compute the ratio MAPD/MAAOQ = 2.5042 and select the nearest value of the ratio from the Table 1 as 2.5236. The values of i and n_2 corresponding to the ratio 2.5236 and MAPD = 0.0015 are i = 2846, n_2 = 23 and f = $1/n_2$ = 1/23 = 0.04. Thus i = 2846, k = 2846, f = 0.04 are the parameters selected for the mixed sampling plan having CSP-3 plan with k = i as attribute plan for a specified MAPD = 0.0015 and MAAOQ = 0.000599.

Practical Application of Mixed Sampling Plan with CSP-3(k=i) plan as attribute Plan

The cooking oil is expected to contain fat, vitamin E etc., in a specified proportion. Inspection of such oil means testing one or more characteristics of the product. Here Mixed Sampling Plan with CSP-3(k=i) as attribute plan can be used as a basis for acceptance or rejection of such items. The characteristic to be inspected is the "vitamin E" of the item for which there is a specified lower limit of 33.5g specified by the producer with a known standard deviation (σ) of 0.2g.

In this example, L = 33.5g, σ = 0.2g and k = 1.5.

$$A = L + K\sigma = 33.5 + (1.5)(0.2) = 36.5g.$$

Now, by applying the variable inspection first, take a random sample of size n_1 = 10 and find the sample average $\bar{X}$ of the characteristic. If $\bar{X} \geq$ 36.5g accept the units otherwise apply the attribute inspection. Under attributes inspection, by taking CSP-3(k=i) as attribute plan, for the specified MAPD = 0.0015 (15 non-conformities out of 10000 units) and MAAOQ = 0.000599 (599 non-conformities out of 1000000 units) the value of i = 2846, and n_2 = 23,

Step l: Specify f = 0.04 and i = 2846.

Step 2: Begin 100% inspection.

Step 3: After i = 2846 units in succession have been found without a defective, start sampling inspection.

Step 4: Randomly inspect a fraction f = 0.04 (4 out of 100 units)of the units.

Step 5: When a defective is found, inspect the next 4 units, if an additional defective is found revert to 100% inspection otherwise, continue sampling for k = 2846 successive sample units. If no defectives are found in k = 2846, continue sampling by selecting a fraction f = 0.04 of the units. If a defective is found in the k = 2846 samples revert to 100% inspection immediately.

To reduce the number of parameters from three to two, it is assumed k=i. So CSP-3(k=i) is a particular type of sampling plan of CSP-3 with k=i.

At the end of the time t switch back to variable sampling plan for the units produced after the time t.

The OC curve for the plan i = 2846 and n_2 = 23 is shown in Figure 1:

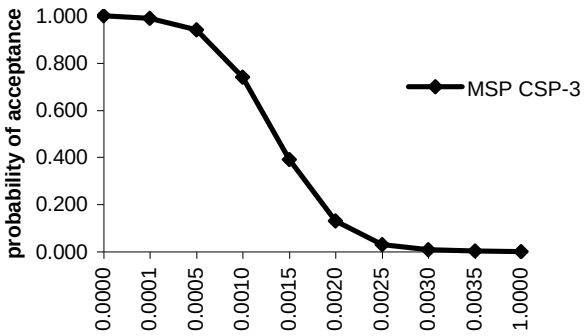

Figure 1: OC curve for i=2846, n_2=23

6. Conclusion:

In this paper a procedure for the construction and selection of independent mixed sampling plan having CSP-3(k=i) as attribute plan is presented. A table is also provided for the easy selection of the plans when MAPD and MAAOQ are known. Practical application for the

sampling plan is also discussed. If the floor engineers know the levels of MAPD and MAAOQ, they can have their sampling plans on the floor itself by referring to the tables. This provides the flexibility to the floor engineers in deciding their sampling plans. Various plans can also be constructed to make the system user friendly by changing the first stage probabilities (β_*'). Similar plans also be constructed for dependent mixed sampling plan also suggested by Radhakrishnan et.al (2009).

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Table 1: **Various characteristics of Mixed Sampling plan with CSP-3(k=i) plan as attribute plan indexed by MAPD and MAAOQ when $\beta_*' = 0.20$.**

MAPD/ MAAOQ	MAPD in Percent																			
	0.15		0.25		0.4		0.65		1.0		1.5		2.5		4.0		6.5		10.0	
	i	n_2	i	n_2	i	n_2	i	n_2	i	n_2	i	n_2	i	n_2	I	n_2	i	n_2	I	N_2
2.6490	4079	137	2445	137	1523	135	883	96	571	95	371	85	217	77	122	48	67	32	36	17
2.6229	3635	72	2179	72	1335	65	817	64	529	64	345	58	201	53	118	41	66	30	34	14
2.6058	3438	54	1952	41	1218	41	762	45	477	38	338	53	173	26	114	35	64	26	34	14
2.5723	3149	35	1887	35	1178	35	723	35	453	30	300	30	168	23	97	18	60	21	34	14
2.5396	2978	28	1785	28	1114	28	684	28	435	26	289	26	165	22	95	17	60	21	33	13
2.5236	2846	23	1706	23	1075	24	640	21	404	19	275	21	159	19	92	15	58	19	33	13
2.4922	2738	20	1621	19	984	17	595	16	379	15	255	16	151	16	90	14	50	11	33	13
2.4465	2542	15	1523	15	950	15	583	15	377	15	250	15	143	13	86	12	50	11	24	5
2.4316	2525	15	1487	14	927	14	569	14	368	14	244	14	136	11	85	12	47	9	24	5
2.4169	2474	14	1454	13	907	13	545	12	360	13	239	13	132	10	83	11	47	9	24	5
2.3880	2423	13	1421	12	887	12	544	12	352	12	228	11	130	10	78	9	45	8	23	5
2.3460	2290	11	1373	11	856	11	525	11	340	11	225	11	129	10	74	8	42	7	22	5
2.3324	2290	11	1373	11	856	11	511	10	331	10	219	10	121	8	71	7	42	7	23	5
2.3256	2205	10	1322	10	825	10	506	10	318	9	204	8	121	8	71	7	42	7	23	5
2.2857	2138	9	1281	9	799	9	490	9	306	8	185	6	115	7	70	7	42	7	23	5
2.2409	2039	8	1222	8	762	8	467	8	276	6	192	7	114	7	69	7	41	7	22	5
2.1918	1932	7	1158	7	722	7	443	7	286	7	189	7	112	7	68	7	36	5	22	5
2.1563	1817	6	1089	6	679	6	438	7	283	7	154	4	111	7	68	7	36	5	20	4
2.0202	1506	4	902	4	562	4	344	4	222	4	131	3	94	5	53	4	31	4	19	4
1.8691	1262	3	755	3	471	3	288	3	186	3	122	3	81	4	43	3	25	3	15	3
1.7204	1165	3	697	3	434	3	266	3	171	3	113	3	61	3	40	3	23	3	13	3